

# WORKSHOP 2017 SCHEDULE

Wednesday, December 20th. Venue: Aula VI

# Morning

Time	Speaker	Title of the talk
09.00 - 09.15	Registration	
09.15 - 09.45	Opening	
09.45 - 10.15	Serena Dipierro	Long-range phase transitions and minimal surfaces
10.20 - 10.50	Marco Macchia	Enumeration of 2-level polytopes
10.55 - 11.25	Coffee Break	
11.30-12.00	Man Shun Ang	Log-determinant Non-negative Matrix Factorization
		via Successive Trace Approximation
12.05 - 12.35	Gianluca Orlando	Cohesive fracture with fatigue: quasistatic evolution
12.40 - 13.10	Antonio Macchia	Binomial edge ideals of bipartite graphs

## Afternoon

Time	Speaker	Title of the talk
15.30 - 16.00	Edda Dal Santo	Decay Properties of Approximate Solutions for the
		Damped Semilinear Wave Equation on a Bounded
		Unidimensional Domain
16.05 - 16.35	Giuseppe Vacca	Introduction and some recent advances on the
		Virtual Element Mehtod

Thursday, December 21st. Venue: Aula VI

# Morning

Time	Speaker	Title of the talk
09.45 - 10.15	Eugenia Loiudice	Canonical fibrations of $(k, \mu)$ -spaces
10.20 - 10.50	Marco Berardi	Richards' equation for modelling the infilitration into
		unsaturated soils: a focus on handling discontinuities
10.55 - 11.25	Coffee Break	
11.30-12.00	Biagio Cassano	Self-adjointness for the Dirac operator with
		Coulomb-type potentials
12.05 - 12.35	Fabio Difonzo	Control theory and optimization applied to industrial
		problems



## WORKSHOP 2017 BOOK OF ABSTRACTS

Man Shun Ang Log-determinant Non-negative Matrix Factorization via Successive Trace Approximation

Non-negative matrix factorization (NMF) is the problem of approximating a nonnegative matrix X as the product of two smaller nonnegative matrices W and H so that X = WH. In this talk, we consider a regularized variant of NMF, with a log-determinant (logdet) term on the Gramian of the matrix W. This term acts as a volume regularizer: the minimization problem aims at finding a solution matrix W with low fitting error and such that the convex hull spanned by the columns of W has minimum volume. The logdet of the Gramian of W makes the columns of W interact in the optimization problem, making such logdet regularized NMF problem difficult to solve. We propose a method called successive trace approximation (STA). Based on a logdet-trace inequality, STA replaces the logdet regularizer by a parametric trace functional that decouples the columns on W. This allows us to transform the problem into a vector-wise non-negative quadratic program that can be solved effectively with dedicated methods. We show on synthetic and real data sets that STA outperforms state-of-the-art algorithms.

Marco Berardi Richards' equation for modelling the infilitration into unsaturated soils: a focus on handling discontinuities

The Richards' equation is presented, that is a classical tool for modelling water infiltration into unsaturated soils, approximating the soil is a porous medium. For sake of simplicity, the 1D case has been considered, being the process mainly governed by gravity. We focused on the problem of infiltration into layered soils; this issue has been faced by conveniently handling the equation and reducing it to a Filippov system. Both the case in which the discontinuity depends on the depth, and the one in which the discontinuity depends on the pressure gradient have been considered. Numerical methods are proposed and numerical simulations are performed.

This is a joint work with Fabio Difonzo, Michele Vurro and Luciano Lopez.

Biagio Cassano Self-adjointness for the Dirac operator with Coulomb-type potentials

We investigate the self-adjointess of the Dirac operator  $H := -i\alpha \cdot \nabla + m\beta + \mathbb{V}(x)$ , for  $m \in \mathbb{R}$ , and  $\mathbb{V}(x) \sim \nu/|x|$ : such property is of course fundamental, but, in particular in the case  $\nu \geq 1$ , it is not fully understood. We review the known results and focus on a recent work, where it is treated the potential  $\mathbb{V}(x) = |x|^{-1}(\nu\mathbb{I}_4 + \mu\beta - i\lambda\alpha \cdot x/|x|\beta)$ , for  $\nu, \mu, \lambda \in \mathbb{R}$ . We describe the self-adjoint realizations of this operator in terms of the behavior in the origin of the functions of the domain: main strategy of the proof is the study of the properties of H on the partial wave subspaces, where appropriate trace inequalities and Hardy inequalities are given.

These results are part of a collaboration with Fabio Pizzichillo (BCAM).

Edda Dal Santo Decay Properties of Approximate Solutions for the Damped Semilinear Wave Equation on a Bounded Unidimensional Domain

In this talk we consider an initial boundary value problem for a semilinear hyperbolic PDE system equivalent to the one-dimensional damped wave equation, with reflecting boundary conditions. The aim is to study the asymptotic behavior of a certain class of approximate solutions (that preserve the stationary ones) and to show that these approximations accurately describe the exact solutions. We prove that they converge exponentially fast to the (approximate) stationary solution in any  $L^p$  norm up to an error of order  $\Delta x$ . The rate of convergence depends on the  $L^1$  norm of the damping term.

This is a joint work with Debora Amadori and Fatima Agel (University of L'Aquila).

### Fabio Difonzo Control theory and optimization applied to industrial problems

When it comes to tell an AGV to detect an obstacle and stop when coming across it, and then generate a new trajectory to get to its destination; or when it is necessary to store thousands of items in a big industrial warehouse and daily provide a huge amount of finished product according to specific cost functions every two hours, it is necessary to calibrate the right mathematical tools and to model and develop suitable approaches to correctly describe the problem and find a correct and efficient solution. This talk is about how mathematics is applied in everyday activities to high level and complex problems in optimization and control theory at Code Architects Automation, a fast growing firm based in Santeramo in Colle.

#### Serena Dipierro Long-range phase transitions and minimal surfaces

We discuss some recent results on nonlocal minimal surfaces and their connections with nonlocal phase transitions. In particular, we will consider the "genuinely nonlocal regime" in which the diffusion operator is of order less than 1 and present some rigidity and symmetry results.

#### Eugenia Loiudice Canonical fibrations of $(k, \mu)$ -spaces

We give a geometrical interpretation of the contact metric  $(k, \mu)$  condition. The base spaces of the canonical fibrations of  $(k, \mu)$ -manifolds will be described as a complexification or paracomplexification of the sphere or of the hyperbolic space. Then, we give a new homogeneous representation of contact metric  $(k, \mu)$ -manifolds.

### Antonio Macchia Binomial edge ideals of bipartite graphs

Binomial edge ideals are a noteworthy class of binomial ideals that can be associated with graphs, generalizing the ideals of 2-minors. For bipartite graphs we prove the converse of Hartshorne's Connectedness Theorem, according to which if an ideal is Cohen-Macaulay, then its dual graph is connected. This allows us to classify Cohen-Macaulay binomial edge ideals of bipartite graphs, giving an explicit and recursive construction in graph-theoretical terms. Our result represents a binomial analogue of the celebrated characterization of (monomial) edge ideals of bipartite graphs due to Herzog and Hibi.

#### Marco Macchia Enumeration of 2-level polytopes

A (convex) polytope P is said to be 2-level if for every direction of hyperplanes which is facet-defining for P, the vertices of P can be covered with two hyperplanes of that direction. The study of these polytopes is motivated by questions in combinatorial optimization and communication complexity. We present the first algorithm for enumerating all combinatorial types of 2-level polytopes of a given dimension d, and provide complete experimental results for  $d \leq 7$ . Our approach is inductive: for each fixed (d-1)-dimensional 2-level polytope  $P_0$ , we enumerate all d-dimensional 2-level polytopes P that have  $P_0$  as a facet. This relies on the enumeration of the closed sets of a closure operator over a finite ground set. The experimental evidence hints an interesting behavior for the number of combinatorial types of 2-level d-polytopes and suggests several new research directions.

This is a joined work with A. Bohn, Y. Faenza, S. Fiorini, V. Fisikopoulos, K. Pashkovich.

## Gianluca Orlando Cohesive fracture with fatigue: quasistatic evolution

In this talk I will present a model for cohesive fracture subject to fatigue. I will start by clarifying the meaning of quasistatic evolution and by recalling the main results in the field of variational modelling of Fracture Mechanics. Next I will explain the main differences between brittle fracture and cohesive fracture, both from a modelling and a mathematical perspective, stressing in particular on the phenomenon of fatigue in materials. I will then state a theorem regarding the existence of quasistatic evolutions for a cohesive fracture model which accounts for fatigue. In the remaining time I will sketch the proof, underlining the major mathematical difficulties.

The results presented in this talk have been obtained in collaboration with V. Crismale and G. Lazzaroni.

Giuseppe Vacca Introduction and some recent advances on the Virtual Element Mehtod

The Virtual Element Method (VEM) is a very recent technology introduced in [Beirão da Veiga, Brezzi, Cangiani, Manzini, Marini, Russo, 2012, M3AS] for the discretization of partial differential equations.

The VEM can be interpreted as a novel approach that generalizes the classical Finite Element Method to arbitrary element-geometry. By avoiding the explicit integration of the shape functions that span the discrete Galerkin space and introducing a novel construction of the associated stiffness matrix, the VEM acquires very interesting properties and advantages with respect to more standard Galerkin methods, yet still keeping the same coding complexity. For instance, the VEM easily allows for polygonal/polyhedral meshes with non convex elements; it allows for discrete solutions of arbitrary  $C^k$  regularity, defined on unstructured meshes.

The present talk is both an introduction to the VEM, aiming at showing the main ideas of the method, and a brief look at some recent development. In the first part of the talk we will describe the basics of the method on a simple linear model problem. In the second part, we will present some further advancement, such as the Stokes problem and the VEM for curved domains.

These results are part of a collaboration with L. Beirão da Veiga, F. Dassi, A. Russo.