Xmaths Workshop 2019

Program

Thursday, 19 December

9.00 *Opening.*

9.30 Antonella Falini (Università degli Studi di Bari)

10.05 Angela Martiradonna (CNR-IAC, Bari)

10.40 *Coffee break*

11.10 Federica Caforio (Medical University of Graz)

11.45 Mufutau Rufai (Università degli Studi di Bari)

Friday, 20 December

9.00 *Opening.*

9.30 Maria Colomba Comes (Università degli Studi Tor Vergata, Roma)

10.05 Eugenia Loiudice (Philipps-Universität Marburg)

10.40 *Coffee break*

11.10 Alessandra De Luca (Università degli Studi di Milano-Bicocca)

11.45 Vincenzo Scattaglia (Università degli Studi di Pisa)

12.20 Nico Michele Schiavone (Università degli Studi La Sapienza, Roma)

Lunch

14.30 Giovanni Girardi (Università degli Studi di Bari)

15.05 Felisia Chiarello (INRIA Sophia-Antipolis)

15.40 *Coffee break*

16.10 Maria Elena Griseta (Università degli Studi di Bari)

16.45 Pasquale Cascarano (Università degli Studi di Bologna)

Organizers:
Marco Gallo
Nicola Picoco
Laura Selicato
Caterina Sportelli

Abstracts

Thursday 19, 9.30:

QI based Quadrature Schemes with applications to (Adaptive) IgA-BEMs.

Antonella Falini

Boundary Element Methods (BEMs) are techniques used to numerically solve differential problems by using Boundary Integral Equations (BIEs) [1]. Such BIEs present singular kernels, hence, designing accurate and efficient quadrature schemes is of fundamental importance.

In this talk, quadrature rules based on spline quasi-interpolation (QI) are presented. The proposed schemes result robust, efficient and very accurate when compared to other quadrature rules available in literature [2]. Moreover, due to the local nature of the QI technique, the rules can be easily used for hierarchical splines as well. This allows to treat more challenging differential problems whose exact solution lacks of regularity, i.e. it is singular, or it exhibits strong features, by locally refining exactly the interested regions [3]. We apply the presented quadrature rules to evaluate BIEs derived by 2D Laplace problems in the context of the isogeometric BEMs.

This is a joint work with Aimi, Calabrò, Giannelli, Kanduĕc, Sampoli and Sestini.

References

[1] Sauter S. and Schwab C., *Boundary Element Methods*, vol.39 of Springer Series in Computational Mathematics, Springer-Verlag, Berlin, (2011).

[2] Calabrò F., Falini A., Sampoli M. L., and Sestini A., *Efficient quadrature rules based on spline quasi-interpolation for application to IGA-BEMs.* Journal of Computational and Applied Mathematics, **338**, 153-167, (2018).

[3] Falini A., Giannelli C., Kanduĕc T., Sampoli M. L., and Sestini A., *An adaptive IgA-BEM with hierarchical B-splines based on quasi-interpolation quadrature schemes.* International Journal for Numerical Methods in Engineering, **117**(10), 1038-1058, (2019).

Thursday 19, 10.05:

Optimal control models for invasive species: analysis, methods and applications

Angela Martiradonna

Improving strategies for the control and eradication of invasive species is an important aspect of nature conservation, an aspect where mathematical modelling and optimization play an important role. We introduce some optimal control models for the control of invasive species that aim to find the best temporal or spatiotemporal resource allocation strategy for the population reduction.

We show how the analysis of the state-control optimality system provides a qualitative description of the dynamics and, in some cases, conduces to derive the expression of the optimal solution for the free terminal time problem. For the numerical approximation of the boundary value state-costate Hamiltonian system, we introduce a numerical method based on symplectic Runge-Kutta schemes applied in a forward-backward procedure.

The spatiotemporal formulation of the problem is based on a reaction-diffusion equation with a Holling II reaction term, while a budget constraint is applied to the control. We use the optimal control theory for proving the well-posedness of the problem in a weak formulation.

The model is applied to the case of the invasive plant *Ailanthus altissima* in the Alta Murgia National Park, in support of management decisions. Remote sensing derived input layers are assimilated in the model to estimate the initial species distribution and its habitat suitability, empirically extracted by a land cover map of the study area.

This work has received fundings from the Innonetwork Project COHECO, Regione Puglia.

Thursday 19, 11.10:

In silico-models of the human circulation: towards a digital twin
<u>Federica Caforio</u>

In-silico models of the human health and disease can offer an efficient way to provide new insights into the physiology and the progression of a disease, or to predict the outcome of a treatment. The long-term objective is to create a "digital twin", a personalised in-silico model of the patient. In particular, in the context of cardiovascular applications, the last decades have witnessed a growing interest in reduced-order (one- or zero-dimensional) methods for patient-specific modelling of complex cardiovascular networks. A key-advantage of these models is that they can perform with reasonable accuracy and computational cost, so they are particularly suitable for inverse-problem strategies. These models have been validated against in vitro experiments and in vivo measurements and have proved to provide useful insights for the understanding of cardiovascular physiology and pathology.
The aim of this talk is to give an overview on the mathematical grounds of such models, which are based on Euler's equations, as well as on the numerical methods in use today to translate these models into powerful tools to simulate the human circulation. Moreover, I will describe some clinical applications and discuss further perspectives.

Thursday 19, 11.45:

Last point second-derivative Lobatto type method
for solving efficiently first-order differential problems
<u>Mufutau Rufai</u>

In this talk, I will present a Lobatto type hybrid block method with a second-derivative which is applied to obtain approximate solutions of first-order initial-value problems in ordinary differential equations (ODEs). The new scheme is derived through interpolation and collocation approaches and the characteristics of the obtained method are analyzed. An embedding-like technique is considered and executed in variable step size mode to get better performance of the method. The newly proposed scheme gives numerical solutions to some real-life application problems to determine its efficiency and accuracy. The numerical solutions obtained are robust and found favorably when compared with other methods of the same order existing in the literature.

Friday 20, 9.30:

More than a simple cell trajectory: time-lapse microscopy
and mathematical algorithms to improve the understanding of biological systems
<u>Maria Colomba Comes</u>

The development of the rapid and low-cost techniques for microfluidic device fabrication has highlighted the Organs On Chips (OOCs) as one of the most promising approaches for optimizing therapies and understanding biological mechanisms. The synergic integration of microfluidic devices, time-lapse microscopy and image analysis has attracted attention to the exploitation of the information content contained inside the cell trajectories. Extracted cell motility descriptors may be indeed considered a useful source of information to quantify and to discern relevant biological differences among diverse counterparts of the same scenario.
In this talk, we describe how data analysis, novel mathematical methods and deep learning approaches coupled with TLM can improve the performance and the understanding of different experimental scenarios, such as for example in OOCs, where the visualization of reconstituted complex biological processes such as multi-cell type migration and cell-cell interactions are possible.

Friday 20, 10.05:

On the topology of metric f-K-contact manifolds

Eugenia Loiudice

We observe that the class of metric f-K-contact manifolds, which naturally contains that of K-contact manifolds, is closed under forming mapping tori of automorphisms of the structure. We show that the de Rham cohomology of compact metric f-K-contact manifolds naturally splits off an exterior algebra, and relate the closed leaves of the characteristic foliation to its basic cohomology.

Friday 20, 11.10:

Monotonicity formula
and its applications to the unique continuation property for elliptic problems

Alessandra De Luca

My current research interest is devoted to study the monotonicity formula and some of its applications, for instance the unique continuation property for second order elliptic equations. In particular, I am interested in studying the following perturbed problem

$$\begin{cases} -\Delta u = f(x)u, & \text{in } B_{\bar{R}} \setminus \Gamma, \\ \quad u = 0, & \text{on } \Gamma, \end{cases} \tag{1}$$

for some potential $f \in L^{\infty}_{\text{loc}}(B_{\bar{R}} \setminus \{0\})$, where $B_{\bar{R}}$ is the open ball of $\mathbb{R}^{N+1}$ ($N \geq 2$) for some $\bar{R} > 0$ and Γ is a closed subset of $\mathbb{R}^N$.

A solution of this problem can be approximated by smooth functions with compact support contained in the closure of the ball which vanish in a neighborhood of Γ.

Once we derive a Pohozaev type identity, the aim is to give a precise description of the blow-up limits in order to investigate the order of vanishing at the origin of the solution.

Friday 20, 11.45:

A short tour from the Classical Isoperimetric Problem to Clusters with Density

Vincenzo Scattaglia

The Classical Isoperimetric Problem for single sets and clusters will be introduced, i.e. searching for the optimal set or the optimal partition of the space which minimizes the measure of its boundary once the volumes are fixed.

At this purpose, a crash course on the theory of finite perimeter sets will be exposed.

Later, we will introduce a generalization of the Isoperimetric Problem to volumes and perimeters with a density, presenting some properties of the isoperimetric function and of minimizers, if these exist.

In the end, we will introduce the analogous problem for Clusters, and discuss the results obtained so far.

Friday 20, 12.20:

Localization estimates for eigenvalues of non-self-adjoint Dirac operators

<u>Nico Michele Schiavone</u>

In recent times, non-self-adjoint operators and their spectral properties are attracting more and more attention. In particular, our interest is focalized on the Dirac operator perturbed by a possibly non-Hermitian potential V and on the study of localization estimates for its discrete eigenvalues in dependence of norms of V. Moreover, we research smallness condition on V such that the spectrum of the perturbed operator is the same of the operator with null potential. In this talk we present an overview of the literature on the matter and a result obtained in collaboration with Piero D'Ancona and Luca Fanelli (Sapienza University of Rome).

Friday 20, 14.30:

Critical regularity of nonlinearities in semilinear classical damped wave equations.

<u>Giovanni Girardi</u>

Let us consider the Cauchy problem for the semilinear damped wave equation

$$u_{tt} - \Delta u + u_t = h(u), \qquad u(0, x) = \phi(x), \qquad u_t(0, x) = \psi(x),$$

where $h(s) = |s|^{1+\frac{2}{n}} \mu(|s|)$. Here n is the space dimension and μ is a modulus of continuity. Our goal is to find sharp conditions on μ to obtain a threshold between global (in time) existence of small data solutions (stability of the zero solution) and blow-up behavior even of small data solutions.

Friday 20, 15.05:

Non-local conservation laws for traffic flow modeling

<u>Felisa Chiarello</u>

In this talk, we provide mathematical traffic flow models with non-local fluxes and adapted numerical schemes to compute approximate solutions to such kind of equations. More precisely, we consider flux functions depending on an integral evaluation of the conserved variables through a convolution product.

First of all, we prove the well-posedness of entropy weak solutions for a class of scalar conservation laws with non-local flux arising in traffic modeling. This model is intended to describe the reaction of drivers that adapt their velocity with respect to what happens in front of them. We approximate the problem by a Lax Friedrichs scheme and we provide some estimates for the sequence of approximate solutions. Stability with respect to the initial data is obtained.

We study also the limit model as the kernel support tends to infinity. We also prove the existence for small times of weak solutions for non-local systems in one space dimension, given by a non-local multi-class model intended to describe the behavior of different groups of drivers or vehicles. We present some numerical simulations illustrating the behavior of different classes of vehicles and we analyze two cost functionals measuring the dependence of congestion on traffic composition. Furthermore, we propose alternative simple schemes to numerically integrate non-local multiclass systems in one space dimension. We obtain these schemes by splitting the non-local conservation laws into two different equations, namely, the Lagrangian and the remap steps. We show some numerical simulations illustrating the efficiency of the L-AR schemes in comparison with classical first and second order numerical schemes.

Finally, we introduce a traffic model for a class of non-local conservation laws at road junctions. Instead of a single velocity function for the whole road, we consider two different road segments, which may differ for their speed law and number of lanes.

Friday 20, 16.10:

Distributions for non symmetric weakly monotone position operators
Maria Elena Griseta

We study the vacuum distribution, under an appropriate scaling, of a family of partial sums of non-symmetric position operators on the weakly monotone Fock space. We preliminary show that any single operator has the vacuum law belonging to the free Meixner class. After establishing some relations between the combinatorics of Motzkin and Riordan paths, we give a recursive formula for the vacuum moments of the law of any finite sum. Since the operators are monotone independent, the distribution is the monotone convolution of the free Meixner law above. We also investigate the asymptotic measure for these sums, which turns out to be the sum of an atomic and an absolutely continuous part (w.r.t. the Lebesgue measure), both explicitly computed.
This is a joint work with V. Crismale and J. Wysoczański.

Friday 20, 16.45:

When did Maths and Images start dating?
A gentle introduction to Image Super Resolution in a Variational Framework
Pasquale Cascarano

The resolution of an image is mainly concerned with the number of pixels per inch the image possesses. A low-resolution (LR) frame is an image with a small pixel density and a poor signal content, an high-resolution (HR) frame is an image with a high pixel density and therefore more details. The Super Resolution is a technique that enhances the image resolution and makes the images clearer for human as well as for machines for better information extraction. High Resolution images are required in many fields such as satellite, thermal and medical image processing, multimedia industry, video enhancement and so on. A Super Resolution problem is the task of reconstructing an High Resolution image using either one or multiple LR frames representing the same scene. Single image and Multiple image Super Resolution can be viewed as linear inverse problems, namely the processes of calculating from a set of observations the causal factors that produced them, which are notoriously challenging ill-posed problems in the sense of Hadamard, that is they could not have a solution, or a solution exists but it is not unique, or the solution's behavior doesn't change continuously with the initial conditions. In this seminar we will briefly see how a degradation model of LR frames can be developed using some linear operators such as the *blur* and the *sub-sampling* operators. Once the degradation model has been fixed, the Super Resolution problem can be solved in a variational framework by thinking of the reconstructed HR image as the minimum of a certain cost functional. This functional is simply the sum of two terms: the *fidelity term*, usually an L_1 or L_2 norm of the fixed model residuals, which contains all the information held by the initial LR frames, and a *regularization term* which induces a-priori information on the solution. We will use a Bayesian Framework to inspect how this functional come from a maximum a-posteriori estimation (MAPE) of the unknown HR image and we will underline the importance of two different kind of regularization terms: Total Variation regularizer (TV), suited for images with sharp edges, and L_0-gradient regularizer (L_0), suited for sparse gradient images. Since the Super Resolution problem has been traced back to an optimization problem we need algorithms capable to minimize smooth or no-smooth functions and convex or no-convex functions. Two Super Resolution algorithms that solve the L_2 + TV and the L_2 + L_0 models with a forwardbackward splitting (FBS) and an alternating direction method of multipliers (ADMM), respectively, will be presented and some results with real thermal images and natural images will be shown and compared with some state-of-art Super Resolution methods.