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Contents

Abstracts

A matter of choice

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University of Urbino Carlo Bo

The aim of this talk is to highlight the importance of choosing the right set of assumptions to tackle a problem. Not the simplest, not the easiest to deal with, not the most general, but the right one. We will do this by going through some results of [2] and [1], which deal with symmetry(-breaking) results for solutions of elliptic partial differential equations (PDEs). More precisely, I will introduce some techniques of shape optimization, as well as some Morse theory, and show how they apply to the study of the interplay between the geometry of solutions of PDEs and the geometry of the domains where they are defined. We will see that our choice of assumptions not only served its original purpose better than other possible ones, but also shed light on the path to follow for further discoveries. Based on joint work with A. Iacopetti and F. Pacella.

[1] Afonso, D. G. Semilinear equations in bounded cylinders: Morse index and bifurcation from one-dimensional solutions. J. Math. Anal. Appl. 543, 2 (2024), 128918.

[2] Afonso, D. G., Iacopetti, A., and Pacella, F. Energy stability for a class of semilinear elliptic problems. J. Geom. Anal. 34, 75 (2024).

Finite Energy solutions for nonlinear elliptic equations with competing gradient, singular and $\mathbf{L}^1$ terms

Francesco Balducci

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In this presentation we deal with the following boundary value problem

$$\begin{cases} -\Delta_p u + g(u)|\nabla u|^p = h(u)f & \text{in } \Omega, \\ u \geq 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

in a domain $\Omega \subset \mathbb{R}^N$, $(N \geq 2)$, where $1 \leq p < N$, g is positive and continuous function on $[0, \infty)$, and h is a continuous function on $[0, \infty)$ (possibly blowing up at the origins). We show how the presence of regularizing h and g allows to prove existence of finite energy solutions for nonnegative datum f only belonging in $L^1(\Omega)$.

Joint work with Francescantonio Oliva and Francesco Petitta

Generative AI in mathematics education: pre-service teachers' knowledge and implications for their professional development

Maria Lucia Bernardi

University of Bari Aldo Moro

Incorporating technologies with Generative Artificial Intelligence (GenAI) into education requires a shift in teaching methodologies. However, little is known about how pre-service teachers perceive the relevance and challenges of this incorporation, particularly in mathematics education. This talk investigates pre-service teachers' interactions with GenAI, addressing the relevance and challenges of integrating it in mathematics teaching and discussing possible implications for their knowledge and professional development. Specifically, it intends to understand: How does the pre-service teachers' interaction with GenAI during the design and implementation of teaching activities relate to their professional knowledge? And how does this relation impact the relevance they ascribe to GenAI? In this qualitative and interpretative study, involving seven pre-service mathematics teachers, we analyse the interplay between participants' knowledge and use of a GenAI (in this case, ChatGPT), guided by KTMT (Knowledge for Teaching Mathematics with Technology) model. The main conclusions revealed a landscape characterised by promise and challenge, where GenAI can be a valuable educational tool when used to facilitate discussion and promote critical thinking, highlighting the relevance and development of KTMT. The ability to evaluate and reflect on AI-generated responses can promote professional development, preparing pre-service teachers for an increasing presence of technology in educational environments.

Global hypoellipticity and solvability for a class of PDEs in time-periodic weighted Sobolev spaces

Matteo Bonino

University of Torino

In this talk, we present results on the global hypoellipticity and solvability for a class of evolution operators with coefficient growing polynomially on the space variable which are related to the class of PDEs $Lu = f$, where

$$L = D_t + c(t)\,\mathrm{Op}(p), \quad D_t = -i\partial_t, \quad t \in \mathbb{T} = \mathbb{T}^1, \quad p \in S^{m,\mu}$$

To this end, we introduce a class of Gevrey time-periodic weighted Sobolev spaces, defined as the class of smooth functions $u : \mathbb{T}^n \to \mathscr{S}'\left(\mathbb{R}^d\right)$ that satisfy

$$\sup_{\gamma\in\mathbb{Z}_+^n} \left\{ C^{-|\gamma|}(\gamma!)^{-\sigma} \sup_{t\in\mathbb{T}^n} \|\partial_t^\gamma u(t)\|_{H^{r,\rho}(\mathbb{R}^d)} \right\} < +\infty$$

where $H^{r,\rho}$, known as Sobolev-Kato spaces, are given by

$$H^{r,\rho} = \left\{ u \in \mathscr{S}'\left(\mathbb{R}^d\right) : \|u\|_{r,\rho} = \|\langle x\rangle^r \langle\xi\rangle^\rho u\|_{L^2} < \infty \right\}.$$

We will then present the strategy based on a spectral decomposition approach using the eigenfunctions $\{\phi_j\}_{j\in\mathbb{N}}$ of an elliptic operator $\mathrm{Op}(p)$, by which we obtain an expansion of the form

$$u(t) = \sum_{j\in\mathbb{N}} u_j(t)\phi_j,$$

which allows us to study the equation $Lu = f$ by reducing it to a sequence of ODEs. Finally, we establish necessary and sufficient conditions on the operator L for global hypoellipticity and

solvability, emphasizing their connection with the so-called Diophantine approximation. This is a joint work with F. de Ávila Silva and S. Coriasco.

Learning Soil Moisture Dynamics: From Time-Series Models to Physics-Informed Neural Networks

Mariateresa Bruni

CNR

Soil water content is a fundamental variable for understanding and modeling hydrological processes in agricultural environments, particularly under water stress and soil salinity conditions. This study analyzes high-frequency soil water content time series collected at an experimental agricultural site in Gallipoli, Italy, integrated with meteorological data and irrigation information characterized by different temporal resolutions.

The aim of this work is to investigate neural networks as universal function approximators for modeling soil water content dynamics, following a pathway from purely data-driven approaches to models that explicitly incorporate physical knowledge. In a first stage, forecasting is addressed using deep learning models for time series, such as Chronos in a predominantly univariate setting, and multivariate machine learning models, including Random Forest and Gradient Boosting, which exploit exogenous variables such as precipitation, temperature, and irrigation.

The framework is then extended through Physics-Informed Neural Networks (PINNs), where the neural network is constrained by the physical laws governing water flow in soils. In particular, soil moisture dynamics are modeled by embedding the Richards equation into the loss function. The comparison between data-driven and physics-informed approaches highlights how incorporating physical constraints improves solution consistency and generalization capability, demonstrating the potential of PINNs for modeling soil water content dynamics.

Ground state solutions for the polyharmonic nonlinear scalar field equation

Alessandro Cannone

University of Bari Aldo Moro

I'm going to talk about a result on the existence of ground state solutions for the polyharmonic nonlinear scalar field equation $(-\Delta)^m u = g(u)$. Finally, I will show a new polyharmonic logarithmic Sobolev inequality, whose optimizers are related to the ground state solutions of the equation. The seminar is based on joint paper with S. Cingolani (Università degli Studi di Bari Aldo Moro) and J. Mederski (Institute of Mathematics Polish Academy of Sciences).

Elastic Brownian motion with random jumps from the boundary

Fausto Colantoni

Sapienza University of Rome

We study an elastic Brownian motion on smooth domains, where the particle, instead of being killed at the boundary, restarts from a random position inside the domain. We characterize the process through its SDE and generator, and describe its invariant measure and spectrum. We finally see possible applications in statistical mechanics for search problems. This is a joint work with Mirko D'Ovidio.

Topology Beyond Pixels: Topological Machine Learning for Lung Lesion Classification

Serena Grazia De Benedictis

University of Bari Aldo Moro

Data-driven modeling in real-world scenarios frequently involves high-dimensional and heterogeneous data, often affected by limited or unbalanced sample sizes, variable acquisition conditions, and the need for reliable generalization [1]. These challenges are especially evident in medical imaging, where capturing subtle structural and morphological patterns is essential for supporting accurate diagnostic outcomes.

Topological Data Analysis (TDA) has emerged as a rigorous mathematical framework capable of addressing these issues by characterizing the intrinsic shape of data [2]. Through persistent homology (PH) [3], TDA extracts multiscale geometric information that is typically inaccessible to traditional intensity- or texture-based descriptors. Building on these foundations, Topological Machine Learning (TML) [4] integrates topological descriptors into machine learning pipelines, enabling interpretable, data-efficient, and model-agnostic workflows that remain robust even in the presence of strong data variability or imbalance.

The methodological framework adopted in this work leverages a broad spectrum of topological features to encode the intrinsic geometric complexity of biomedical data. After vectorization, these features are combined with classical learning models, forming an analytical pipeline that enhances interpretability and generalization while requiring limited training data.

This approach is applied to the classification of lung lesions in Computed Tomography (CT) scans from the publicly available IQ-OTH/NCCD dataset [5], characterized by marked class imbalance. Experimental results show that the TML framework achieves high classification accuracy and out- performs deep learning methods under comparable conditions. The results highlight how topology-driven representations capture fine morphological variations and improve robustness to data imbalance, contributing to more reliable and explainable decision-making in medical imaging.

This is a joint work with Nicoletta Del Buono, University of Bari Aldo Moro.

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Laplace and Length Spectra for closed hyperbolic surfaces of large volume

Giacomo Gavelli

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Two of the most studied invariants of a closed hyperbolic surface are the Laplace and the Length spectrum. Even though a complete description of these spectra is difficult to obtain in general, one can rely on good asymptotics for counting the number of eigenvalues of the Laplacian (respectively, length of closed geodesics) in a certain window, as this window expands to infinity. Changing the perspective, one can fix an interval and ask for asymptotics on the number of eigenvalues (respectively, length of closed geodesics) in that interval as the volume grows to infinity. This bears the question on how to obtain sequences of surfaces with increasing volume which give good asymptotics. In this talk I will introduce two such notions, namely Benjamini-Schramm and Plancherel convergence, and discuss their relation.

This talk is based on a joint work with C. Kamp.

The Laplace-Beltrami spectrum on Naturally Reductive Homogeneous Spaces

Jonas Martin Henkel

Marburg University, Germany

Calculating the spectrum of the Laplace-Beltrami operator is a fundamental problem in geometric analysis, explicitly solvable mainly for normal homogeneous spaces. This talk addresses the broader class of compact naturally reductive homogeneous spaces. We present a Freudenthal-type formula for the Laplacian's spectrum in this setting. This framework provides a powerful tool to analyze how the spectrum behaves under metric deformations, particularly for canonical variations of normal homogeneous metrics. Applications will demonstrate how these methods, particularly when combined with refined branching techniques (joint work with Leandro Cagliero), lead towards full spectra of 3-(α, δ)-Sasaki manifolds. This talk is based on a joint project with Ilka Agricola.

Beyond the Standard Stability Criterion for Second-Order Sigma Delta Quantization Schemes

Alessandro Lupoli

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In this talk, we examine the stability conditions of 1-bit second-order sigma–delta quantization schemes, which are widely used for the analog-to-digital (A/D) conversion of bandlimited signals. These schemes rely on feedback filters to shape the quantization noise in the frequency domain, thereby achieving high-resolution reconstruction. For schemes of order higher than one, stability has long been a challenging issue: classical conditions are notoriously restrictive, imposing conservative bounds on the amplitude of the input signal or requiring feedback filters of impractically large length.

We show that, for second-order schemes, these stability conditions can be significantly relaxed. In particular, by exploiting the oversampling, one can stably quantize signals with larger amplitude while using feedback filters that are considerably shorter than those prescribed by the standard criteria.

Joint work with Rohan Joy and Felix Krahmer.

Higher-power Harmonic Maps, Instantons and Yang-Mills Theory

Henrik Naujoks

Marburg University, Germany

In my talk I will present the geometric and physical properties of so-called higher-power harmonic maps, a generalization of harmonic maps described by C. Wood and A. Ramachandran in 2023. Due to the algebraic structure of their equations, they have strong similarities with Yang-Mills theory and are therefore particularly suitable for use in physical field theory. After a short introduction, I will explain the coupling of these maps to a gravitational field and the resulting coupled partial differential equations. I will show how explicit solutions of the system can be constructed on Lie groups under symmetry assumptions. If there is enough time, I will present an instanton theory for higher-power harmonic maps: This yields an easier first-order differential equation whose solutions coincide with those of the original equation. In addition to the simpler construction of solutions, this theory also allows an estimation of the energy of these maps via a topological invariant.

On the instability of traveling water waves

Antonio Milosh Radakovic

Scuola Internazionale Superiore di Studi Avanzati

The Water-Waves equation is a nonlinear PDE describing the motion of the free surface at the top of a fluid under the action of gravity, capturing a phenomenon that is both ubiquitous in nature and deeply fascinating due to its rich and complex dynamics. After giving a brief overview of the equation, we will focus on an important family of equilibrium solutions: the periodic traveling waves. I will review several results achieved in recent years on the instability of these solutions under special types of perturbations in the 2D case, and finally present a novel work providing a general description of the 3D instability.

Propagation of Shubin-Sobolev singularities of Weyl-quantizations of complex quadratic forms

Davide Tramontana

University of Bologna

The aim of the talk is to develop the Hörmander microlocal theory in the isotropic framework and use the results obtained to study the propagation of singularities for an evolution problem of the form

$$\begin{cases} \partial_t u + Au = 0, & \text{in} \quad \mathbb{R}^+ \times \mathbb{R}^n, \\ u(0,\cdot) = u_0 \in \mathcal{S}'(\mathbb{R}^n), \end{cases}$$

with $A = \mathrm{Op}^{\mathrm{w}}(a) \in \Psi^2_{\mathrm{iso}}(\mathbb{R}^n)$ the Weyl-quantization of a complex quadratic form on the phase space

$$a : \mathbb{R}^{2n} \longrightarrow \mathbb{C}, \quad X \longmapsto a(X) = \langle X, QX \rangle,$$

defined by the symmetric complex matrix $Q \in \mathsf{M}_{2n}(\mathbb{C})$ with $\mathrm{Re}\, Q \geq 0$. For such a problem we will obtain a propagation of singularities result in terms of the s-isotropic wave front set, that is we will study the Shubin-Sobolev microsingularities of the solution with respect to those of the initial datum.

Euclidean random matrices for collective photon emission by cold atomic ensembles

Viviana Viggiano

University of Bari Aldo Moro

Photon emission by a cold atomic ensemble is a collective effect: it differs profoundly from emission by a collection of isolated atoms. For instance, the decay rate can be smaller or larger compared to that of a single isolated atom, giving rise to the phenomena of subradiance and superradiance, respectively. In this talk, I will consider an ensemble of cold atoms whose positions are independent and identically distributed according to a three-dimensional Gaussian probability density function. To obtain information on the collective decay modes of this system, I will investigate the spectral properties of a family of Euclidean random matrices, called decay rate matrices, whose entries depend on the Euclidean distance between pairs of atomic positions. In particular, I will focus on the asymptotic regime with an infinite number of atoms in the low-density limit, and show that in this regime the only relevant parameter is the so-called cooperativeness parameter, which quantifies the number of atoms that cooperate coherently in photon emission. For different values of the cooperativeness parameter, I will analyze the microscopic spectral statistics, level spacing, and eigenvectors corresponding to the bulk of the spectrum of the decay rate matrices. I will show that, although these random matrices are Euclidean, the bulk of their spectra exhibits the universal behavior of chaotic quantum systems, characterized by level repulsion and delocalized eigenvectors, and can therefore be described using standard random matrix theory [1].

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